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Electrostatic Probe Electron Current Collection in the Transition Regime

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Introduction

TALBOT and Chou[1] recently developed an approximate analytical analysis, based on the limited results of the rigorous Chou-Talbot-Willis[2] theory, which permits the calculation of the effect of collisions on the probe ion or electron current in the electron retarding field region. Their analytical results have been found to compare favorably with the Kaegi-Chin[3] and Dunn-Lordi[4] ion current measurements and with the Kirchhoff-Peterson-Talbot[5] ion and electron current measurements, all taken using negatively biased cylindrical probes operating in the transition regime. However, the influence of collisions on the electron current attracted to a positively biased probe is also of practical interest. It is therefore the intent of this work to extend the Talbot-Chou[1] approximate analysis to describe the influence of collisions on the electron current collected by a positively biased cylindrical or spherical probe.

In the Chou-Talbot-Willis[2] collisional analysis certain integrals appeared which, because of their complexity, were only evaluated over a small range of the parameters of interest in order to establish the trend of the results. However the applications of these results for negatively biased probes were extended by Talbot and Chou[1] to include the entire range of parameters. This was accomplished by using the Bernstein-Rabinowitz[6] and Laframboise[7] results and the Su-Lam[8] and Cohen[9] results to evaluate these integrals in the collisionless and continuum limits respectively and then using an interpolation formula to span the transition regime between these limits. The same general procedure may be followed for positively biased probes.

One can show that the *rigorous* analytical expressions needed to evaluate the collisionless limits of these integrals for positive probe potentials may be obtained from the Talbot-Chou[1] results for negative probes by interchanging (the notation is that of Ref. 1) n_e and n_i, j_e and j_i and by everywhere replacing $\tau\chi$ by $(-\chi)$ and χ by $(-\tau\chi)$. Similarly the *rigorous* equations for the positive probe valid in the continuum limit may be recovered by interchanging j_e and j_i; K_e and K_i, ξ_e and ξ_i, and by everywhere replacing τ by $1/\tau$. However certain algebraic approximations are also used in the evaluations of these integrals and the resulting probe currents. Obviously the analogous expressions for the positive probe can not be determined by direct substitution from the negative probe results of Talbot and Chou[1] and will therefore be discusssed in detail in the following paragraphs.

Spherical Probe

First consider the spherical case where for positive probe potentials the integral is of the form $J = \int_0^1 \exp(\chi)dz$. In order to evaluate the potential distribution $\chi(z)$ in the collisionless limit it is necessary to have an algebraic representation for the probe current ($j_{e,\infty}$). To be consistent, an expression similar in form to that used for ion current in Ref. 1 is desired. Fortunately Laframboise's[7] numerical results for monoenergetic electrons can be represented within 5% by

$$j_{e,\infty} = f_s(\tau)(\chi_p/\tau\chi_s)\alpha_s \tag{1}$$

provided $f_s(\tau)$ is defined as

$$f_s(\tau) \equiv 1 - \pi\chi_s/4 = j_{e,\infty}z_s^2 \tag{2}$$

Representative values of f_s, $\tau\chi_s$ are shown on Table 1 while α_s is given on Fig. 1. It follows that the potential distribution, which must vary continuously throughout the region $z_s \leq z \leq 1$ and must satisfy Eqs. (1) and (2) at the probe and absorption radii, respectively, may be approximated by

$$j_{e,\infty}z^2 = f_s(\tau)(\chi/\chi_s)^{\beta_s} \tag{3}$$

where

$$\beta_s = \alpha_s[1 - \ln\tau/\ln(\chi_p/\chi_s)] \tag{4}$$

The above integral may then be numerically evaluated. For electron collection the contribution from the outer integral (over the region $0 \leq z \leq z_s$) is of the form $J_{e,\infty}^{(1)} = G_1(\tau)/j_{e,\infty}^{1/2}$ while the inner integral ($z_s \leq z \leq 1$) may be taken, with only a small loss in accuracy, as $J_{e,\infty}^{(2)} = \tau^{1/2}G_2(\xi)$. Representative values of G_1 and G_2 are given in Table 1 and Fig. 1 respectively. For monoenergetic ion collection the integral analogous to $J_{e,\infty}^{(2)}$ is negligible. However, recall that the amount of electric field that penetrates past the sheath edge is nearly proportional to the thermal energy of the repelled species. Therefore for large τ the ion shielding at positive probe potentials is nearly complete and the contribution from the inner integral may equal that from the outer. It follows that the electron current attracted to a spherical probe operating in the transition regime is given by,

$$j_{e,\infty}/j_e = 1 + (1 + K_e)^{-1}[G_1(\tau)j_{e,\infty}^{1/2} + \tau^{1/2}j_{e,\infty}G_2(\xi)] + K_e^{-1}(1 + K_e)^{-1}(1 + \tau^{-1})^{-1}j_{e,\infty}z_0 \tag{5}$$

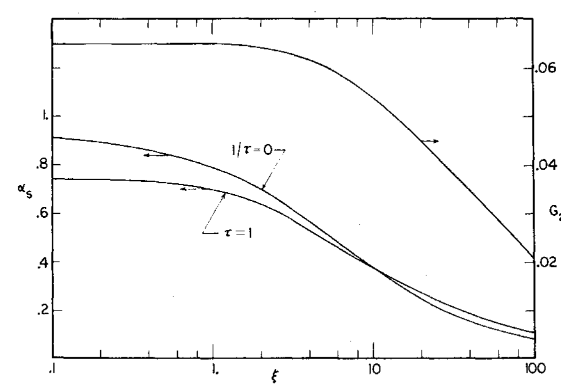

Fig. 1 Functions for evaluation of spherical probe current.

Table 1 Functions for the evaluation of the probe current

	$\tau = 1$	$\tau = 2$	$\tau = 5$	$\tau = 10$
$-\tau\chi_s$	0.52	0.59	0.65	0.67
$f_s(\tau)$	1.41	1.23	1.10	1.06
$G_1(\tau)$	1.07	1.05	1.03	1.02
$f_c(\tau)$	1.38	1.20	1.08	1.04

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Cylindrical Probe

The formulation for the cylindrical probe parallels that for the spherical probe with the exception that the normalized continuum limit probe current has been approximated by

$$j_{e,0} = K_e[(1 + \tau)/\tau]/\ln(l/r_p x_0) \tag{6}$$

to remain consistent with the Kirchhoff-Peterson-Talbot[5] results. In the collisionless limit Laframboise's numerical results for cylindrical probes collecting monoenergetic electrons may be closely reproduced by

$$j_{e,\infty} = f_c(\tau)(\chi_p/\tau\chi_c)^{\alpha_c} \tag{7}$$

provided $f_c(\tau)$ is defined as

$$f_c(\tau) \equiv (4/\pi)^{1/2}\left(\frac{\pi}{4} - \chi_c\right)^{1/2}$$

Representative values of $f_c(\tau)$ are given in Table 1, α_c is shown on Fig. 2 and $\chi_c = -(\ln 2)/\tau$. It follows that the potential distribution in the region $z_c \leq z \leq 1$ may be approximated by

$$j_{e,\infty}z = f_c(\tau)(\chi/\chi_c)^{\beta_c} \tag{8}$$

where

$$\beta_c = \alpha_c[1 - \ln\tau/\ln(\chi_p/\chi_c)] \tag{9}$$

and, the inner integral is then

$$J_{e,\infty}{}^{(2)} = \beta_c[E_1(\chi_c) - E_1(\chi_p)] \tag{10}$$

The electron current attracted to a cylindrical probe operating in the transition regime is then

$$\begin{aligned} j_{e,\infty}/j_e = 1 + (1 + K_e)^{-1}\{G_1(\tau)j_{e,\infty}{}^{1/2} + \\ j_{e,\infty}\beta_c[E_1(\chi_c) - E_1(\chi_p)]\} + \\ j_{e,\infty}K_e{}^{-1}(1 + K_e)^{-1}(1 + \tau^{-1})^{-1}\ln(l/r_pX_0) \end{aligned} \tag{11}$$

where $j_{e,\infty}$ is the normalized value for the cylindrical probe current in the collisionless limit.

Results and Discussion

The collisional effects were generally found to become more severe as the relative thermal energy of the collected species increased. For example at large values of τ and χ_p, the normalized cylindrical probe electron current density is significantly less than the normalized ion current density when $K_e < 10$.

Typical values of the cylindrical probe electron current are shown in Fig. 3 for values of $\chi_p = |\chi_f| + 10$ where χ_f is the probe floating potential. No solution exists for the current collected by a cylindrical probe operating in the continuum limit and the expression used here is just an approximation. The results for the cylindrical probe are, therefore, thought to be most accurate for large Knudsen numbers and should be used with caution for small K where the solution is dominated by the continuum limit value for the probe current.

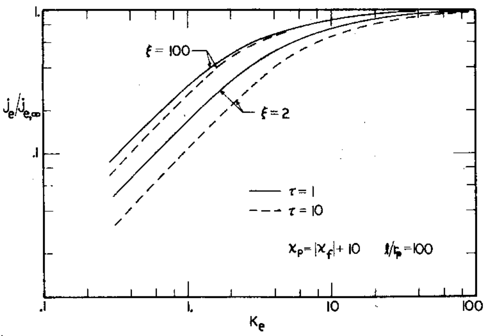

Fig. 3 Cylindrical probe electron current in the transition region.

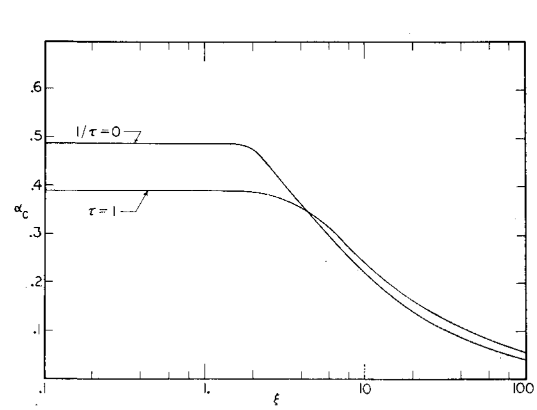

Fig. 2 Correlation exponent for electron collection by a cylindrical probe.

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Some Measurement in Pipe Flow

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IN semibounded or in-bounded flow, the structure of turbulence largely depends on the properties of the so-called viscous layer in the vicinity of the boundary. The observations of Kline and co-workers[1,2] in turbulent boundary layers have shown some interesting features contradicting the widely accepted model of the viscous sublayer which has been well-known for some time. The salient features of the previous postulation are that the so-called viscous sublayer close to the wall is three-dimensional and is dominated by vigorous bursts

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